Estimation of the Lifting Condensation Level

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Introduction

The lifting condensation level (LCL) is the height at which an air parcel undergoing adiabatic ascent reaches saturation. This level is easily determined on a Skew-T Log-P diagram (AWSM, 1969). Stull (2000) presents a simple linear relationship between LCL height and dewpoint depression,

$$z_{LCL} = aD \tag{1}$$

where z_{LCL} represents the LCL height in kilometers; $a = 0.125$ km °C^{-1}, a constant; and D is the dewpoint depression ($T - T_d$) in degrees Celsius. In order for this approximation to be implemented correctly, the reader should be refreshed of its origin. This note derives the equation from simple thermodynamic principles and comments on the statistical basis for the value chosen to represent the constant *a*.

Derivation

Assuming a constant dry adiabatic lapse rate with height, the temperature at any level during dry adiabatic ascent can be written as

$$T(z) = T_0 - \Gamma_d z\,, \tag{2}$$

where T_0 is the initial temperature, Γ_d is the dry adiabatic lapse rate, and z is the vertical displacement. If we assume that the change in *dewpoint* is also constant with height (i.e., the layer through which the parcel ascends is well-mixed), a version of (2) can be utilized to calculate the dewpoint at any level,

$$T_{dew}(z) = T_{dew0} - \Gamma_{dew} z\,, \tag{3}$$

where T_{dew0} is the initial dewpoint temperature (at the same level as T_0), and Γ_{dew} is the "dewpoint lapse rate." Note that, although the same "lapse rate" principle is applied to temperature and to dewpoint, the two lapse rates are *not* equal. Realizing that an air parcel's temperature and dewpoint are equal at the LCL, we can write the height of displacement as

$$z = \frac{T_0 - T_{dew0}}{\Gamma_d - \Gamma_{dew}}\,. \tag{4}$$

If the reference level (i.e., the level at which T_0 and T_{dew0} are taken) is assumed to be at the surface, and the displacement is to the LCL, then z represents the LCL height above the surface.

Derivation of the dry adiabatic lapse rate will be avoided since it is fundamentally known that $\Gamma_d = g/C_{pd} \approx 9.8$°C/km. The calculation of Γ_{dew} presents a more interesting dilemma. If e is the partial vapor pressure that corresponds to dewpoint T_d, then the following relationship holds:

$$e_{(at\ LCL)} = e(T_{dew})\,. \tag{5}$$

Differentiating (5) with respect to height gives

$$\frac{de}{dz} = \frac{d}{dz}\left(e(T_{dew})\right) = \frac{de}{dT_{dew}}\frac{dT_{dew}}{dz}\,. \tag{6}$$

The partial vapor pressure e can be written as (see, e.g., Wallace and Hobbs, 1977)

$$e = \frac{wp}{w+\varepsilon}\,, \tag{7}$$

where w is the mixing ratio, p is the pressure, and ε is the ratio of the gas constants for dry air to that for water vapor ($\varepsilon = R_d/R_v = 0.622$). For unsaturated adiabatic ascent, mixing ratio is assumed to be conserved (this is the same procedure used on a Skew-T Log-P diagram), and as such, the left-hand side of (6) can be rewritten as

$$\frac{de}{dz} = \frac{w}{w+\varepsilon}\frac{dp}{dz} = \frac{e}{p}\frac{dp}{dz}\,. \tag{8}$$

We now write our equality as

$$\frac{1}{p}\frac{dp}{dz} = \frac{1}{e}\frac{de}{dT_{dew}}\frac{dT_{dew}}{dz}\,. \tag{9}$$

A useful form of the Clausius-Clapeyron equation (Tsonis 2002, p. 99) is

$$\frac{de}{dT_{dew}} = \frac{l_v e}{R_v T_{dew}^2}\,,\ \text{or}\ \frac{de_s}{e_s} = \frac{l_v dT}{R_v T^2} \tag{10}$$

since the C-C Equation is an expression of *equilibrium* between the vapor and liquid phase. This form neglects the volume of water vapor compared to the volume of dry air and takes the latent heat of vaporization, l_v, as a constant.

The air parcel ascends adiabatically, so Poisson's equation can be used to relate temperature and pressure for the parcel:

$$Tp^{\frac{1-\gamma}{\gamma}} = \text{constant}\,,\ \text{or}\ \frac{1}{T}\frac{dT}{dz} = \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{p}\frac{dp}{dz} \tag{11}$$

after logarithmic differentiation with respect to height. In (11), γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume for a dry air parcel. In (11), note that *dT/dz* is simply the negative of the dry adiabatic lapse rate (remember that Poisson's relations are for an adiabatic process!), and utilize (10) and (11) to simplify (9), to yield the result

$$-\left(\frac{\gamma}{\gamma-1}\right)\frac{g}{C_{pd}T} = \frac{l_v}{R_v T_{dew}^2}\frac{dT_{dew}}{dz}. \tag{12}$$

Although the specific heats for moist air are dependent upon moisture content, they can be replaced (to an error less than 0.25% for common atmospheric conditions) by the specific heats for dry air. The left-hand side then reduces to

$$-\left(\frac{\gamma}{\gamma-1}\right)\frac{g}{C_{pd}T} = -\frac{g}{R_v T}. \tag{13}$$

Recalling that $R_d/R_v = \varepsilon$, it is now possible to rewrite (12) in terms of the dewpoint lapse rate, in a simplified form:

$$-\frac{dT_{dew}}{dz} = \Gamma_{dew} = \frac{g}{\varepsilon l_v}\frac{T_{dew}^2}{T}. \tag{14}$$

This is the expression for the change in dewpoint with height. Note that the value is dependent upon both temperature *and* dewpoint (i.e., it is valid only at a single, given combination of temperature and dewpoint).

Discussion and Final Result

Data from the Integrated Global Radiosonde Network (IGRA; Durre et al. 2006) was examined at the Oklahoma City/Norman, Oklahoma site, from 1957-2000, for soundings with CAPE of at least 1000 J kg^{-1}. Some 1995 soundings were found, with the following surface statistics:

Temperatures 13.3 to 39.3°C;
Dewpoints 10.0 to 25.2°C; and
Dewpoint depressions 0.7 to 25.3°C.

These values set reasonable bounds on expected surface conditions in midlatitude convective environments. Dewpoint lapse rates were then calculated for temperatures from 14°C to 39°C and dewpoint temperatures from 10°C to 25°C. Any value in the matrix where the dewpoint depression is zero (or less) or greater than 25 is excluded. In this domain, the average dewpoint depression is 1.74°C km^{-1}. Given that convective environments infrequently have dewpoints less than 10°C (where the dewpoint lapse rate is lower), we can state that dewpoint can be estimated to decrease by approximately 1.8°C per kilometer (this could be proven by refining the matrix and taking another average). The decrease becomes greater as surface dewpoint increases. On a Skew-T Log-P diagram, it appears that the dewpoint changes at a constant rate with height, since the mixing ratio lines are not curved. However, temperature lines are slightly curved, so Γ_{dew} changes slowly with height.

Since

$$\Gamma_d - \Gamma_{dew} = 9.8 \,{}^{\circ}C/km - 1.8 \,{}^{\circ}C/km = 8 \,{}^{\circ}C/km, \tag{15}$$

Stull's formula (2000, given here as Eq. 1) is probably a good estimation of the height of the lifting condensation level.

Assumptions and Caveats

The equation derived here relies heavily on the following assumptions:

(a) vertical motion below the LCL is adiabatic;
(b) the layer from the surface to the LCL is well mixed;
(c) the specific heats for dry air are equal to those for moist air;
(d) the latent heat of vaporization is constant; and
(e) over the given range of temperatures, the lapse rate of dewpoint is negligible.

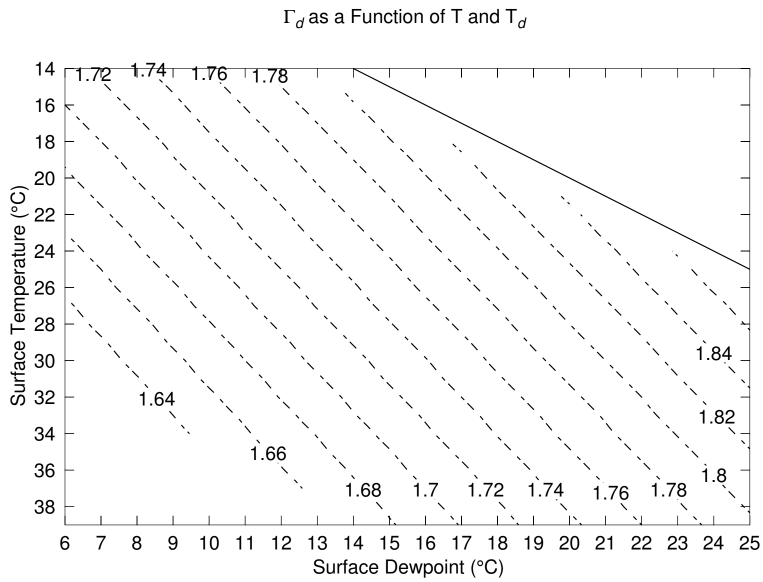

Figure 1. Dewpoint lapse rate (Γ_{dew}; C km^{-1}) as a function of surface dewpoint and temperature. Note the unphysical values (i.e., below ground level) at the upper right, where $T_d > T$.

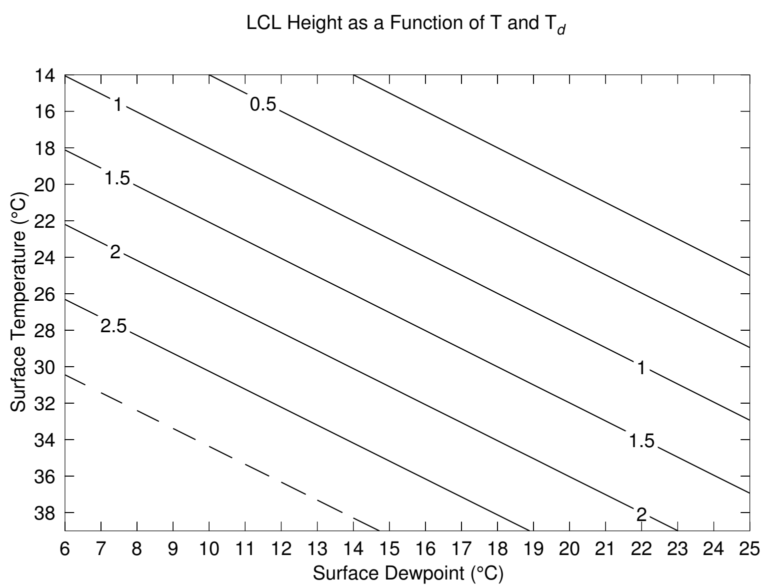

Figure 2. LCL height (km) as a function of surface dewpoint and temperature. Contours every 0.5 km, up to 3 km. Unphysical values as in Fig. 1.

References

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